Inter-Risk Diversification Effects between Credit and Market Risk

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Context

Challenges for Integrated Models of Market and Credit Risk

- Common time horizon for credit and market risk: long (Multi-step models of market risk including optimal rebalancing strategies)
- Credit risk models with stochastic interest rates
- Market risk changes default probabilities, exposure at default, recovery rates
- Credit events change market exposure
- Joint distribution of credit and market (and macro) risk factors at long time horizon
- Portfolio valuation as a joint function of credit and market risk factors

Our contribution

- The Orthodoxy
  (Rosenberg & Schuermann, Duffie & Pan, Singleton, Dimakos & Aas, Kuritzkes & Schuermann & Weiner, Walder):

  "Integrated risk is often larger than pure market risk or pure credit risk. But because of imperfect correlations it is always smaller than the sum of pure market and pure credit risk."
- We show:
  $$\rho_{M+C}(P) > \rho_M(P) + \rho_C(P)$$
  is possible
- Distinguish: This effect is not related to some risk measures violating sub-additivity w. r. t. portfolios

The Benchmark Portfolio

- $n_l = 1000$ long and $n_s = 1000$ short zero coupon bonds maturing in 10 years

$$P(r, PD, n_d) = (n_l - n_s)F\frac{1 - PD}{(1 + r)^{10}} - n_d F(1 - RR)$$

RR: recovery rate as a percentage of face value
F: face value of one bond
r: p.a. risk free interest rate
n_d: number of defaulted long bonds (binomial from n_l experiments with success probability *PD*)
PD: risk-neutral probability of default over the bond life

- Conditional on PDs, defaults of individual bonds are assumed to be independent of each other

Risk Factor Models

- market risk: r (10 yr default free interest rate)
- credit risk:
  PD default probability (systematic)
  n_d: actual number of defaults (ideosyncratic)
- No recovery risk: RR assumed constant
- Model of marginals: AR(1)+GARCH (1,1)-EVT
- Model of copula: Student

Risk Measurement Approaches

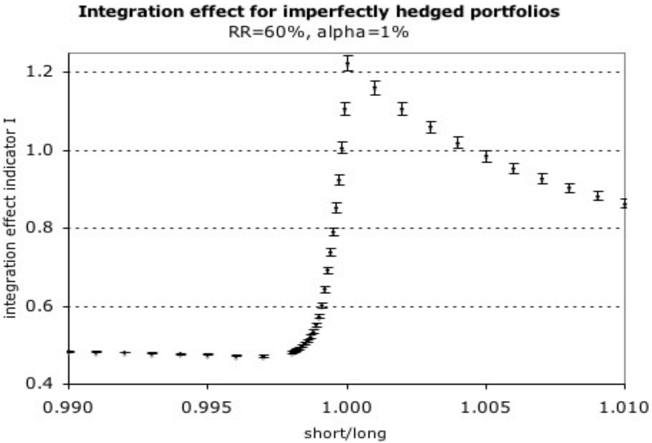
- Method (0): Simulate P/L-dist from changes of r only, assuming PD equal zero.
  Just Market Risk
- Method (1): Simulate P/L-dist from changes of PD only, assuming interest rate to be constant.
  Just Credit Risk
- Method (2): Simulate P/L-dist from simultaneous changes of interest rate r and PD.
  Integrated Credit&Market Risk

Results on Benchmark Portfolio

ES α	(0) MR no CR	(1) CR no MR	(2) Integrated MR&CR	(2)/((0)+(1)) diversification effect I
0.10 %	0	978.4	1294.1	1.323
0.25 %	0	819.9	1049.2	1.280
0.50 %	0	707.0	883.5	1.250
1.00 %	0	599.5	734.0	1.224
2.50 %	0	465.6	554.5	1.191
5.00 %	0	369.0	429.9	1.165
10.00%	0	277.3	314.6	1.135

Negative Inter-Risk Diversification Effect between CR and MR:
Risk cap for integrated MR & CR > risk cap for CR + risk cap for MR by 13–32%

Results for Partially Hedged Portfolios



Negative Diversification Effect occurs for portfolios
almost perfectly hedged against market risk.
Separate Risk Management is dangerous if you follow Golden Rule of ALM

Concentration of credit risk

ES, $\alpha = 1\%$ long cntpt's	(0) MR no CR	(1) CR no MR	(2) Integrated MR&CR	(2)/((0)+(1)) diversification effect *I*
1	0	4 240	4 570	1.078
5	0	4 203	4 558	1.084
10	0	3 963	4 314	1.088
30	0	1 968	2 260	1.148
50	0	1 422	1 664	1.169
100	0	1 081	1 256	1.162
500	0	663	805	1.213
1 000	0	599	734	1.224
5 000	0	540	670	1.240
10 000	0	531	661	1.244

With increasing number of counterparties,
the relative importance of credit risk is reduced,
the negative diversification effect becomes stronger.

Dependence on Recovery Rate

ES, $\alpha = 1\%$ RR	(0) MR no CR	(1) CR no MR	(2) Integrated MR&CR	(2)/((0)+(1)) diversification effect *I*
0	0	11 515	11 649	1.012
0.1	0	9 492	9 626	1.014
0.2	0	7 521	7 654	1.018
0.3	0	5 617	5 750	1.024
0.4	0	3 799	3 931	1.035
0.5	0	2 096	2 227	1.063
0.6	0	598	730	1.224

With increasing recovery rate,
the relative importance of credit risk is reduced,
the negative diversification effect becomes stronger.

Dependence on Copula

ES Student c.	(0) MR no CR	(1) CR no MR	(2) Integrated MR&CR	(2)/((0)+(1)) diversification effect *I*
0.10 %	0	978.4	1294.1	1.323
0.25 %	0	819.9	1049.2	1.280
0.50 %	0	707.0	883.5	1.250
1.00 %	0	599.5	734.0	1.224
2.50 %	0	465.6	554.5	1.191
5.00 %	0	369.0	429.9	1.165
10.00%	0	277.3	314.6	1.135
Gaussian c.				
0.10%	0	1 498	1 597	1.066
0.25%	0	1 205	1 287	1.068
0.50%	0	1 019	1 087	1.067
1.00%	0	855	907	1.061
2.50%	0	664	698	1.051
5.00%	0	531	552	1.039
10.00%	0	402	412	1.025

Negative Diversification Effect is stronger for Student copula than for Gauss copula.
CR and MR&CR are larger for Gauss than for Student copula.

Dependence on risk measure

ES	(0) MR no CR	(1) CR no MR	(2) Integrated MR&CR	(2)/((0)+(1)) diversification effect *I*
0.10 %	0	978.4	1294.1	1.323
0.25 %	0	819.9	1049.2	1.280
0.50 %	0	707.0	883.5	1.250
1.00 %	0	599.5	734.0	1.224
2.50 %	0	465.6	554.5	1.191
5.00 %	0	369.0	429.9	1.165
10.00%	0	277.3	314.6	1.135
VaR				
0.10%	0	802.7	1 007.8	1.255
0.25%	0	653.7	798.6	1.222
0.50%	0	549.5	657.6	1.197
1.00%	0	448.9	526.8	1.174
2.50%	0	323.8	368.0	1.137
5.00%	0	232.3	256.8	1.105
10.00%	0	149.7	155.4	1.038

Negative Diversification Effect is stronger for ES than for VaR.

And here are the main points again ...

Negative inter-risk diversification effects are possible:
risk cap for integrated MR&CR > risk cap for CR + risk capital for MR

- may be serious, but only for almost perfectly hedged portfolios
- is more serious for higher recovery rates
- is more serious for portfolios with well diversified credit risk
- larger for Student than for Gaussian copula
- is more serious for ES than for VaR

The negative diversification effect is more dangerous
when risk measurement and management techniques
are more elaborate.