

# Regulatory Capital for Market and Credit Risk Interaction: Is Current Regulation Always Conservative?

Thomas Breuer

Martin Jandačka

Klaus Rheinberger

Martin Summer

Conference IMCR, Berlin 6. and 7.12.2007



## Introduction

Integrated versus separate analysis of market and credit risk

A toy example of risk underestimation

A real world example

Conclusions



# Outline

## Introduction

Integrated versus separate analysis of market and credit risk

A toy example of risk underestimation

A real world example

Conclusions



# Regulation of Market and Credit Risk: The traditional view.

- ▶ In the work of the Basel Committee there has been a tradition to distinguish market from credit risk and to treat both categories independently in the calculation of risk capital.

# Regulation of Market and Credit Risk: The traditional view.

- ▶ In the work of the Basel Committee there has been a tradition to distinguish market from credit risk and to treat both categories independently in the calculation of risk capital.
- ▶ Leaving aside operational risk, Pillar 1 of Basle II requires separate regulatory capital for market and credit risk:

$$RC_c + RC_m \quad (1)$$

# Regulation of Market and Credit Risk: The traditional view.

- ▶ In the work of the Basel Committee there has been a tradition to distinguish market from credit risk and to treat both categories independently in the calculation of risk capital.
- ▶ Leaving aside operational risk, Pillar 1 of Basle II requires separate regulatory capital for market and credit risk:

$$RC_c + RC_m \quad (1)$$

- ▶ In this paper we argue that this approach is problematic and that it can lead to significant underestimation of risk.

# The Traditional View in Practice

- ▶ Regulators have traditionally thought of credit risk as mainly relevant for the banking book and market risk as mainly relevant for the trading book. The regulatory categorization follows the traditional department organization of banks.

# The Traditional View in Practice

- ▶ Regulators have traditionally thought of credit risk as mainly relevant for the banking book and market risk as mainly relevant for the trading book. The regulatory categorization follows the traditional department organization of banks.
- ▶ The **rough association** of different risk categories to banking and trading book may have inspired the widely held view that current regulation as expressed in equation (1) is conservative.



# The Argument for Diversification Benefits between Market and Credit Risk

**Premise 1** 'Diversification': Under a subadditive risk measure the risk of the total portfolio will be smaller or at most equal to the sum of the risk of the banking book and of the trading book.

**Premise 2** Credit risk is just relevant to the banking book and market risk is just relevant to the trading book.

**Conclusion** Under all subadditive risk measures total risk will be smaller or at most equal to the sum of market risk and credit risk.

This argument is valid.

We criticize Premise 2 and the Conclusion.



# The Structure of the Argument

- ▶ We show: **Only if** premise 2 holds the conclusion will be true.

# The Structure of the Argument

- ▶ We show: **Only if** premise 2 holds the conclusion will be true.
- ▶ If the portfolio is not separable along the categories market and credit risk the Basle Approach may **underestimate** the true risk.

## Related Research

- ▶ Literature that takes a critical view on the traditional risk categorization and concentrating on modelling issues: Jarrow and Turnbull [2000], Medova and Smith [2005], Barnhill and Maxwell [2002].

## Related Research

- ▶ Literature that takes a critical view on the traditional risk categorization and concentrating on modelling issues: Jarrow and Turnbull [2000], Medova and Smith [2005], Barnhill and Maxwell [2002].
- ▶ Literature that does not take issue with the traditional categorization but looks at portfolios subject to market- and credit risk as subportfolios of the overall bank portfolio: Rosenberg and Schuermann [2006] and Dimakos and Aas [2004].

# Outline

Introduction

**Integrated versus separate analysis of market and credit risk**

A toy example of risk underestimation

A real world example

Conclusions



# Integrated and separate analysis of market- and credit risk

- ▶ Current regulation is conceptually based upon the distinction between market and credit risk.

# Integrated and separate analysis of market- and credit risk

- ▶ Current regulation is conceptually based upon the distinction between market and credit risk.
- ▶ **Market risk** is defined as the risk that a financial position changes its value due to the change of an underlying market risk factor, like a stock price, an exchange rate or an interest rate.



# Integrated and separate analysis of market- and credit risk

- ▶ Current regulation is conceptually based upon the distinction between market and credit risk.
- ▶ **Market risk** is defined as the risk that a financial position changes its value due to the change of an underlying market risk factor, like a stock price, an exchange rate or an interest rate.
- ▶ **Credit risk** is defined as the risk of not receiving the promised payment on an outstanding claim.

# A formalization of separated and integrated risk analysis

Assume some separation of risk factors into **market risk factors** described by a vector  $e \in E$  and **credit risk factors**, described by a vector  $a \in A$  is given.

The **value of the portfolio** in dependence of  $a$  and  $e$  is given by the function  $v : A \times E \rightarrow \mathbb{R}$ .

# A formalization of separated and integrated risk analysis

- ▶ **Market risk** is defined as the value change of a portfolio which arises from moves in market risk factors, assuming that credit risk factors are constant at some  $a_0$ :

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0).$$

# A formalization of separated and integrated risk analysis

- ▶ **Market risk** is defined as the value change of a portfolio which arises from moves in market risk factors, assuming that credit risk factors are constant at some  $a_0$ :

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0).$$

- ▶ **Credit risk** deals with value changes caused by moves in credit risk factors, assuming all market risk factors are constant at  $e_0$ :

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0).$$

# A formalization of separated and integrated risk analysis

- ▶ **Market risk** is defined as the value change of a portfolio which arises from moves in market risk factors, assuming that credit risk factors are constant at some  $a_0$ :

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0).$$

- ▶ **Credit risk** deals with value changes caused by moves in credit risk factors, assuming all market risk factors are constant at  $e_0$ :

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0).$$

- ▶ **Integrated risk** is the value change caused by simultaneous moves of market and credit risk factors:

$$\Delta v(a, e) := v(a, e) - v(a_0, e_0).$$



# Approximation of regulatory capital by adding up categories

In a separate analysis of market and credit risk regulatory capital for market and credit risk are simply added up. This amounts to assuming that integrated risk is approximately the sum of market risk plus credit risk:

$$\Delta v(a, e) \approx \Delta c(a) + \Delta m(e).$$

This corresponds to the approximation

$$v(a, e) \approx v(a_0, e_0) + \Delta c(a) + \Delta m(e) =: \hat{v}(a, e).$$

## Approximation errors can go both ways

Our main message is that the approximation  $\Delta c(a) + \Delta m(e)$  **not always overestimates** but **sometimes underestimates** the true integrated  $\Delta v$ . If in some scenario  $(a, e)$  the approximation error

$$\begin{aligned}d(a, e) &:= \Delta v(a, e) - \Delta c(a) - \Delta m(e) \\ &= v(a, e) - \hat{v}(a, e)\end{aligned}$$

is negative, we have a **malign risk interaction**. If  $d$  is non-negative in all scenarios, we have a **benign interaction** effect.

# A classification of portfolios where the approximation is exact

## Proposition

*The approximation is exact, that is  $\Delta v(a, e) = \Delta c(a) + \Delta m(e)$ , if and **only if**  $v$  has the form*

$$v(a, e) = v_1(a) + v_2(e).$$

*In this case the portfolio is separated into two subportfolios, one depending only on credit risk factors, the other depending only on market risk factors.*



# Going from value change functions to risk measures and risk capital

- ▶ The properties of the value change functions in various scenarios  $(a, e)$  carry over to risk measures and risk capital. If the parameter space  $A \times E$  is equipped with a probability measure, the functions  $\Delta v, \Delta c, \Delta m$  give rise to random variables.

# Going from value change functions to risk measures and risk capital

- ▶ The properties of the value change functions in various scenarios  $(a, e)$  carry over to risk measures and risk capital. If the parameter space  $A \times E$  is equipped with a probability measure, the functions  $\Delta v, \Delta c, \Delta m$  give rise to random variables.
- ▶ To these random variables one can apply any risk measure  $\rho$ .

# Measuring the effects of integrated risk analysis

We measure the integration effect by two indices:

$$I := \rho(\Delta c) + \rho(\Delta m) - \rho(\Delta v)$$
$$I_{rel} := \frac{\rho(\Delta v)}{\rho(\Delta c) + \rho(\Delta m)}$$

$I$  gives the EUR amount by which the sum of risk capital for market risk plus risk capital for credit risk exceeds risk capital for integrated risk.  $I_{rel}$  measures the percentage amount by which total risk exceeds the sum of market and credit risk.

# Positive and negative integration effects

## Proposition

*In the case of positive interaction of risk ( $d \geq 0$ ) inter-risk diversification is positive for all coherent risk measures  $\rho$ :*

$$\rho(\Delta v) \leq \rho(\Delta c) + \rho(\Delta m).$$

*Otherwise, in the case of negative interaction of risk ( $d < 0$  somewhere), there exists a coherent risk measure  $\rho$  for which inter-risk diversification of risk capital is negative:*

$$\rho(\Delta v) > \rho(\Delta c) + \rho(\Delta m).$$

# Outline

Introduction

Integrated versus separate analysis of market and credit risk

**A toy example of risk underestimation**

A real world example

Conclusions



## Foreign Currency Loans: A toy example

- ▶ A single obligor has taken a Swiss Franc loan of 1 Euro. At the current exchange rate of  $f(0)$  this amounts to a swiss franc loan of  $1/f(0)$ . After one year the loan expires and the payment obligation is  $o := f(1)/f(0) =: e$ .

## Foreign Currency Loans: A toy example

- ▶ A single obligor has taken a Swiss Franc loan of 1 Euro. At the current exchange rate of  $f(0)$  this amounts to a swiss franc loan of  $1/f(0)$ . After one year the loan expires and the payment obligation is  $o := f(1)/f(0) =: e$ .
- ▶ We assume that the market risk factor  $e$  can vary in the interval  $[0, \infty)$  and that the interest rate is zero.

## Foreign Currency Loans: A toy example

- ▶ A single obligor has taken a Swiss Franc loan of 1 Euro. At the current exchange rate of  $f(0)$  this amounts to a swiss franc loan of  $1/f(0)$ . After one year the loan expires and the payment obligation is  $o := f(1)/f(0) =: e$ .
- ▶ We assume that the market risk factor  $e$  can vary in the interval  $[0, \infty)$  and that the interest rate is zero.
- ▶ Without further specifications assume that the obligor's EUR payment ability at the expiry of the loan is  $a$ , and that this credit risk factor  $a$  can vary in the interval  $[0, \infty)$ .



## Foreign Currency Loans: A toy example

- ▶ The **portfolio value** in this example is

$$v(a, e) := \min(a, o) - o = -\max(o - a, 0) = -\max(e - a, 0).$$

## Foreign Currency Loans: A toy example

- ▶ The **portfolio value** in this example is

$$v(a, e) := \min(a, o) - o = -\max(o - a, 0) = -\max(e - a, 0).$$

- ▶ **Credit risk** is then

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0) = -\max(e_0 - a, 0) + \max(e_0 - a_0, 0).$$

## Foreign Currency Loans: A toy example

- ▶ The **portfolio value** in this example is

$$v(a, e) := \min(a, o) - o = -\max(o - a, 0) = -\max(e - a, 0).$$

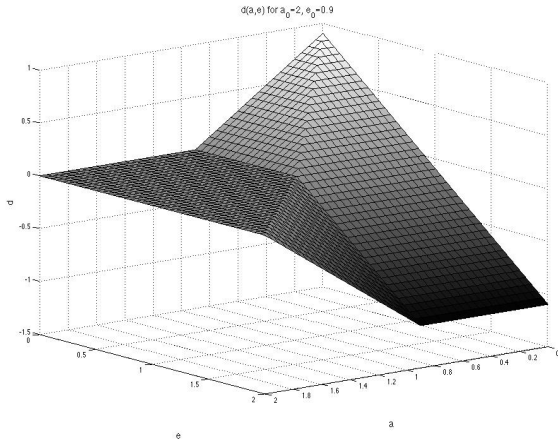
- ▶ **Credit risk** is then

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0) = -\max(e_0 - a, 0) + \max(e_0 - a_0, 0).$$

- ▶ **Market risk** is

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0) = -\max(e - a_0, 0) + \max(e_0 - a_0, 0).$$

# Credit risk and market risk interaction



# Outline

Introduction

Integrated versus separate analysis of market and credit risk

A toy example of risk underestimation

**A real world example**

Conclusions



# Can risk underestimation matter in a real world example?

- ▶ Portfolio of foreign currency loans with  $N$  obligors indexed by  $i = 1, \dots, N$ , one period.

## Can risk underestimation matter in a real world example?

- ▶ Portfolio of foreign currency loans with  $N$  obligors indexed by  $i = 1, \dots, N$ , one period.
- ▶ Customer's payment obligation to the bank at time 1 in home currency is

$$o_i = l_i(1 + r) f(1)/f(0) + l_i s f(1)/f(0)$$

# Can risk underestimation matter in a real world example?

- ▶ Portfolio of foreign currency loans with  $N$  obligors indexed by  $i = 1, \dots, N$ , one period.
- ▶ Customer's payment obligation to the bank at time 1 in home currency is

$$o_i = l_i(1 + r) f(1)/f(0) + l_i s f(1)/f(0)$$

- ▶ Let obligor  $i$ 's payment ability be  $a_i$ . The profit bank makes with obligor  $i$  is

$$v_i := \min(a_i, o_i) - l_i(1 + r)f(1)/f(0).$$



## A broad brush approach to model payment ability

The payment ability of obligor  $i$  is distributed according to

$$a_i(1) = a_i(0) \cdot \frac{GDP(1)}{GDP(0)} \cdot \epsilon,$$
$$\log(\epsilon) \sim N(\mu, \sigma)$$

where  $m$  and  $a(0)$  are constants, and  $\mu = -\sigma^2/2$  ensuring  $E(\epsilon) = 1$ . For different obligors the realizations of  $\epsilon_i$  are independent.

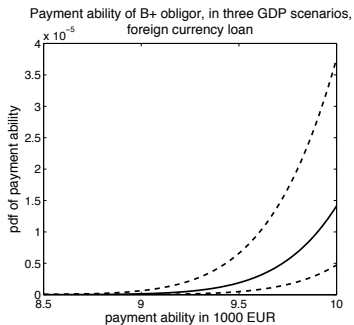
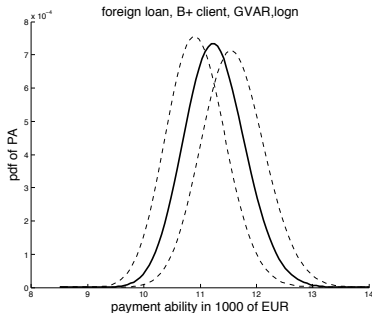
# Calibrating the idiosyncratic risk distribution

Let  $p_i$  be the conditional probability that obligor  $i$  defaults given macroeconomic and market variables take their expected value.

$$p_i = P[a_i < o_i | o_i = E(o_i), GDP(1) = E(GDP(1))].$$

Since the distribution of  $\epsilon_i$  has only one free parameter  $\sigma$  it can be determined by this calibration condition.

# Payment ability distribution under GDP shifts



## Modelling the probability law of risk factors

- ▶ We model the joint dynamics of risk factors by means of a GVAR model due to Pesaran, Schuermann and Weiner [2006].

## Modelling the probability law of risk factors

- ▶ We model the joint dynamics of risk factors by means of a GVAR model due to Pesaran, Schuermann and Weiner [2006].
- ▶ The GVAR is a vector error correction model capturing economic interdependence between countries or regions.

## Modelling the probability law of risk factors

- ▶ We model the joint dynamics of risk factors by means of a GVAR model due to Pesaran, Schuermann and Weiner [2006].
- ▶ The GVAR is a vector error correction model capturing economic interdependence between countries or regions.
- ▶ The basic idea is to set up for each region a VECM model with foreign variables entering the equation as weakly exogenous. Foreign variables are constructed as (trade weighted) averages across foreign countries or regions.

## Modelling the probability law of risk factors

- ▶ We model the joint dynamics of risk factors by means of a GVAR model due to Pesaran, Schuermann and Weiner [2006].
- ▶ The GVAR is a vector error correction model capturing economic interdependence between countries or regions.
- ▶ The basic idea is to set up for each region a VECM model with foreign variables entering the equation as weakly exogenous. Foreign variables are constructed as (trade weighted) averages across foreign countries or regions.
- ▶ This construction allows the separate estimation of country models. These separately estimated models can then be "stacked" consistently in a global model. The weak exogeneity assumption permits a "stacking-procedure" that can use the parameter estimates of the individual country models.

# Portfolio and Monte Carlo Simulation

- ▶  $N = 100$  loans of  $l_i = 10\,000$  EUR taken out in CHF by customers in the rating class B+ ( $p_i = 2\%$ ), or in rating class BBB+ ( $p_i = 0.1\%$ ).



## Portfolio and Monte Carlo Simulation

- ▶  $N = 100$  loans of  $l_i = 10\,000$  EUR taken out in CHF by customers in the rating class B+ ( $p_i = 2\%$ ), or in rating class BBB+ ( $p_i = 0.1\%$ ).
- ▶ Bank extends loans only to customers with  $a_i(0)$  equal to 1.1 times the loan amount. Spread of 100 bp above the LIBOR.

# Portfolio and Monte Carlo Simulation

- ▶  $N = 100$  loans of  $l_i = 10\,000$  EUR taken out in CHF by customers in the rating class B+ ( $p_i = 2\%$ ), or in rating class BBB+ ( $p_i = 0.1\%$ ).
- ▶ Bank extends loans only to customers with  $a_i(0)$  equal to 1.1 times the loan amount. Spread of 100 bp above the LIBOR.
- ▶ Distribution of  $v$  (cf. eq. (24)) calculated by a Monte Carlo simulation of 100 000 draws from the distribution of market and macro risk factors  $f(1)$ ,  $GDP(1)$ , and  $r$ . In each macro scenario defaults of the customers' payment abilities were determined by draws from the distribution of the payment ability process.

# Expected Shortfall of different risks for the foreign currency loan portfolio.

Rating	$\alpha$	MR only	CR only	Integrated	$I$	$I_{rel}$
<i>B+</i>	10%	652	130	6403	-5607	8.19
<i>B+</i>	5%	760	206	11321	-10340	11.72
<i>B+</i>	1%	971	377	31693	-30330	23.52
<i>B+</i>	0.1%	1202	613	83462	-81631	45.97
<i>B++</i>	10%	652	977	11697	-10068	7.18
<i>B++</i>	5%	760	1247	17612	-15604	8.77
<i>B++</i>	1%	791	1881	37707	-34855	13.22
<i>B++</i>	0.1%	1202	2799	85032	-81031	21.25

# Outline

Introduction

Integrated versus separate analysis of market and credit risk

A toy example of risk underestimation

A real world example

**Conclusions**



# Conclusions

- ▶ The traditional approach of treating market and credit risk separately in current regulation is problematic because many portfolios are not separable along these categories.

# Conclusions

- ▶ The traditional approach of treating market and credit risk separately in current regulation is problematic because many portfolios are not separable along these categories.
- ▶ Current practice of determining regulatory capital is conservative only if such subportfolios can be constructed.

# Conclusions

- ▶ The traditional approach of treating market and credit risk separately in current regulation is problematic because many portfolios are not separable along these categories.
- ▶ Current practice of determining regulatory capital is conservative only if such subportfolios can be constructed.
- ▶ If portfolio positions depend simultaneously on both market and credit risk factors a separation into subportfolios of market and credit risk leads to wrong valuation and as a consequence to wrong assessment of the true portfolio risk.

# Conclusions

- ▶ The traditional approach of treating market and credit risk separately in current regulation is problematic because many portfolios are not separable along these categories.
- ▶ Current practice of determining regulatory capital is conservative only if such subportfolios can be constructed.
- ▶ If portfolio positions depend simultaneously on both market and credit risk factors a separation into subportfolios of market and credit risk leads to wrong valuation and as a consequence to wrong assessment of the true portfolio risk.
- ▶ The approach of treating market and credit risk separately for the calculation of regulatory capital should be reconsidered.