Gödel's Theorem and Theories of Everything

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Outline

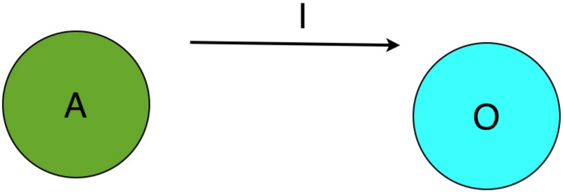
1. Warm-Up: The picture of the picture in the picture
2. Universality and Incompleteness
3. Measurements from inside
4. Implications for Theories of Everything

Jan van Eyck: Wedding of Giovanni Arnolfini, 1434

The picture of the picture in the picture

No picture with finite resolution can contain a perfect picture of itself.

Set-up: Measurements



- A: measurement apparatus, with state space S_A
- O: observed system, with state space S_O
- Measurement: infer information about O-state from A-state. Inference map $I : \mathcal{P}(S_A) \to \mathcal{P}(S_O)$. $I(X)$: the set of all states after the measurement in which O can be if A is in some state in X.
- $I(X) = \bigcup_{s \in X} I(\{s\})$.
- Exact measurability of a state s of O: $\exists S \subset S_A : I(S) = \{s\}$. This implies $\exists s_A \in S_A : I(\{s_A\}) = \{s\}$.

Perfect picture

We have a perfect picture if every state is exactly measurable:

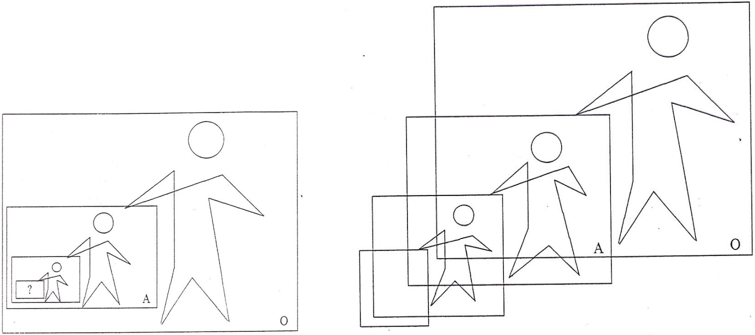
For every state of the object
there exists at least one state of the apparatus
refering just to that state of the object.

For all $y \in S_O$ there is a $x \in S_A$ such that $I(\{x\}) = \{y\}$.

With finite resolution, no picture can properly contain a picture of itself

- $S_A \subset S_O$.
- Strictly contained: $S_A \neq S_O$.
- Finite resolution of A: $\mathrm{card}(S_A)$ finite
- Proposition: In this situation a perfect picture is not possible.
- Argument: If a perfect picture were possible, I would be a surjective map $S_A \to S_O$,
  but there can be no surjective map from the finite proper subset S_A of S_O onto S_O.

The picture of the picture in the picture



Even a picture with finite resolution
can contain a perfect partial picture of itself.

Outline

Tarski and Gödel

Theorem (Gödel)

If

- *rules of inference are defined primary recursively*
- *system is rich enough for arithmetics*
- *system is consistent*

then

- *provability can be formulated <u>in</u> the system*
- *not all propositions are provable or disprovable according to this concept of provability.*

Theorem (Tarski)

In a language rich enough for arithmetics,
a truth predicate which can be formulated <u>in</u> the language cannot be total.

Tarski and Gödel popular: Universality or Completeness

A semantic concept cannot both

- be internal to the theory and
- apply to every object of the theory.

A theory cannot be both

- universal and
- complete.

Analogy but not consequence: The measurement problem of QM

Universal validity of QM

(A) 1-1 relation between physical quantities and self-adjoint operators

(B) Closed systems always evolve according to the Schrödinger equation.

(C) A discrete physical quantity has a well-defined value iff the system is in an eigenstate of the corresponding operator

Completeness: A complete theory contains the semantic concept ‘experiment’

(D) Every single experiment has some (perhaps unknown) result.

Measurement problem: (A), (B), (C), (D) imply a contradiction.

Sketch: The measurement problem of QM

- Want to measure discr. non-deg. observable with eigenvectors $o_1, \ldots, o_n$
- (A): there is a pointer observable P with eigenvectors $p_1, \ldots, p_n$
- time evolution $o_i \otimes p_0 \to o_i \otimes p_i$
- (B): $\sum_i o_i \otimes p_0 \to \sum_i o_i \otimes p_i =: s_1$
- (C), (D): Final state of single system: some $o_i \otimes p_i$
  final state of ensemble: $\sum_i |\lambda_i|^2 |o_i \otimes p_i\rangle\langle o_i \otimes p_i| =: s_2$
- (A): $s_1 \neq s_2$. Thus (A) - (D) imply a contradiction

Lesson: Apparatus cannot both
be described by traditional QM (QM universal)
and account for measurements (QM complete).

Interpretations of Quantum Mechanics

Interpretations of QM can be classified according to which assumption is dropped in order to save (D).

- Drop (A): Bohr, superselection rules
- Drop (B): Neumann's projection postulate, modified dynamics
- Drop (C): modal interpretation, Bohm, many worlds, many minds

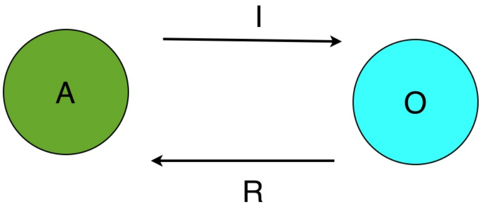
Outline

Measurements from inside

Now: General state spaces S_A, S_O.

- Consider the case where apparatus A is part of the observed system O.
- State of O determines state of A by restriction.
  $R : S_O \to S_A$.
- A is strictly contained in O.
  there are $s, s' \in S_O : s \neq s', R(s) = R(s')$.

Circular reference in measurements from inside



Lemma (Consistency Lemma)

If A is part of O the inference map I must satisfy the following consistency condition:

$$R(I(s_A)) = \{s_A\}$$

for all $s_A \in S_A$.

Measurements from inside cannot be perfect

Theorem

If A is strictly part of O, there are O-states which A cannot distinguish.

Sketch of Proof

O-states s, s' with $R(s) = R(s')$ are indistinguishable:

- If s, s' were exactly measurable there would be s_A, s'_A with $I(s_A) = \{s\}$ and $I(s'_A) = \{s'\}$.
- This would imply
  $\{s\} = I(s_A) = I(R(I(s_A))) = I(R(s)) =$
  $= I(R(s')) = I(R(I(s'_A))) = I(s'_A) = \{s'\}$,
  contradicting $s \neq s'$.

Outline

What do I take as Theory of Everything?

- No part of the material world should be excluded from its domain of validity.
- Assumptions about determinism are neither necessary nor excluded:
  e.g. mental phenomena might or might not be described by a TOE.

Determinism without Predictability

Can we define determinism as predictability for closed systems, from hypothetically precise initial data?

- We know: No observer can measure exactly all states of a system in which she is contained.
- Therefore: It is not possible to perfectly predict the future of a system in which one is contained.
- More: It is not possible to perfectly predict the future of a system in which one is not contained.

Perfect predictability fails with and without determinism.

Is a TOE possible?

- A TOE should be able to describe every possible observer.
- It should be able to describe the universe, which has only internal obersers.
- No observer can distinguish all states of the universe.
- If the union of all observers is properly contained in the universe,
  not even all possible observers in cooperation can achieve this.

For a realist this is not a problem,
but an ardent positivist has to conclude, that a TOE contains some meaningless ‘dead freight’.

Weak TOE

An ardent positivist can embrace the concept of weak TOE, which is universally valid in a relative sense:

A weak TOE can be applied to ANY subsystem, but not to the universe as a whole.