

# Stress Testing Macro Economic Scenarios

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# Outline

- 1 Defining the Extent of Tests
- 2 Enhancing Credit Risk Models with Macroeconomics
- 3 Modelling the Macroeconomic Context
- 4 The Joint Distribution of Risk Factors
- 5 Evaluating Loss Distributions in Macro Scenarios
- 6 Case Study: Macro Stress Testing in the Systemic Risk Monitor



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# Deterministic and Stochastic Scenarios

- *Deterministic scenario:*

A state  $s$  of the world at the time horizon,  
as far it is of relevance to the portfolio value.

“Certainly the state of the world will be  $s$ .”

The value of every risk factor is specified.

- *Stochastic scenario:*

a set  $S$  of states

“The world be in some state in the set  $S$ .”

No further probability information given.

# Generalised and Partial Scenarios

A probability distribution  $s$  over the states of the world at the time horizon, usually not the predicted or true distribution.

- “GDP will grow by 2.5%, all other variables are distributed conditionally on this.”
- “Interest rates will be up by 75 bp, all other variables distributed uniformly.”

# Traditional Stress Tests

## What is it?

Current state of the market:  $s_{CM}$

Hence, current portfolio value:  $P(s_{CM})$ .

Performing stress tests:

- 1 Select scenarios  $s_{\text{stress1}}, s_{\text{stress2}}, \dots$   
(according to some criterion)
- 2 Calculate portfolio values  $P(s_{\text{stress1}}), P(s_{\text{stress2}}), \dots$
- 3 Compare these values with  $P(s_{CM})$ .

Stress test = scenario analysis



# Traditional Stress Tests

## How to select scenarios

### How to select scenarios

- Standard scenarios
- Historical scenarios
- Subjective worst case scenarios



# Dangers of Traditional Stress Tests

## An Example

For a sample portfolio of equities:  
Stress tests with standard and historical scenarios  
may nourish a false illusion of safety

Scenario	Relative Loss	Plausibility
Worst DPG	- 11%	once in 10 yrs.
Black Friday	- 4%	once in 19 yrs.
Worst Case	<b>-18%</b>	<b>once in 8 yrs.</b>





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## Dangers of Traditional Stress Tests

- A stress scenario for one portfolio might be a lucky strike for another portfolio
- Stress tests with standard and historical scenarios may nourish a false illusion of safety
- Subjective worst case scenarios are often too implausible to trigger management action

# Dangers of Traditional Stress Tests

**But:**

Stress Tests can be the basis of informed risk decisions ...

- ... if the scenarios are plausible
- ... if we are confident that there are no scenarios which are worse and more plausible

# Maximum Loss

$AD$ : The scenarios with prob density above a certain level.  
“Scenarios above a certain minimal relevance threshold”  
(For normal or t-dist  $AD$  is an ellipsoid.)

$$\text{MaxLoss}_{AD}(P) := \max_{s \in AD} (P(s_{CM}) - P(s))$$

“Above the plausibility threshold nothing worse than MaxLoss can happen.”

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# Macroeconomics matters for credit risk

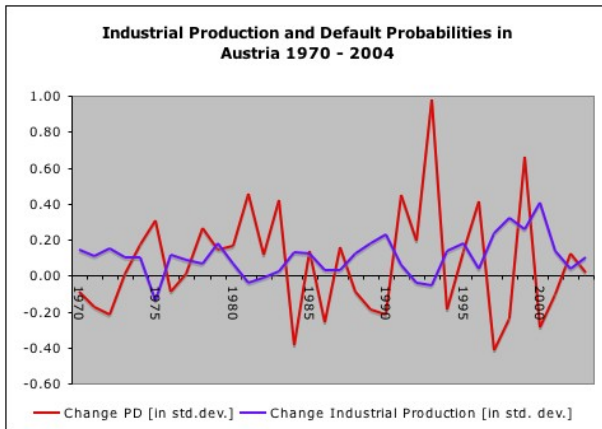
Many ingredients of credit risk models depend on the macroeconomic situation

- default rates and other rating transition probabilities
- recovery rates
- exposure at default  
(depends often on interest rates, which are partially macro)





# Macroeconomics and Credit Risk



# Macroeconomic Risk Factors in Credit Risk Models

## The task

Include macroeconomic variables in credit risk models

- dependence of PDs (and transition probabilities) on macroeconomic variables
- dependence of exposure at default and collaterals on interest rates, equity indices etc, which are partially macro



# Example 1: Credit Portfolio View (Wilson/McKinsey, Nickell et al.)

$$P_{j,t} = \frac{1}{1 + \exp(-Y_{j,t})}$$

$$Y_{j,t} = \beta_{j,0} + \beta_{j,1}X_{j,1,t} + \beta_{j,2}X_{j,2,t} + \dots + \beta_{j,m}X_{j,m,t} + \nu_{j,t}$$

$P_{j,t}$ : cond. PD for obligor in country/industry  $j$

$Y_{j,t}$ : macroeconomic index for country/region  $j$

$\beta_j$ : coefficient vector to be estimated from default data in country/region  $j$

$X_{j,t}$ : vector of macro variables for country/region  $j$   
evolution modelled by an autoregressive model

$\nu_{j,t}$ : error term



## Example 2: Structural default models with macroeconomic variables (Pesaran, Schuermann)

- structural model: firm (equity) value below threshold triggers default
- model firm (equity) value as

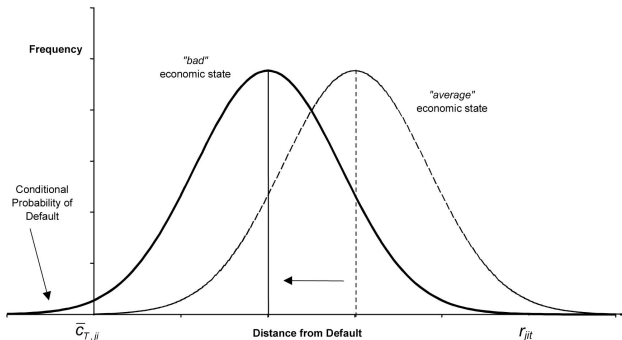
$$r_{ij,t} = \mu_{ijt} + \xi_{ij,t}$$

$\mu_{ijt}$ : forecastable conditional mean,  
depending on macro variables, firm specific 'alphas',  
regional fixed and time trend effects, global exogeneous variables

$\xi_{ij,t}$ : error distribution, involving firm shocks,  
macro shocks, exogeneous shocks



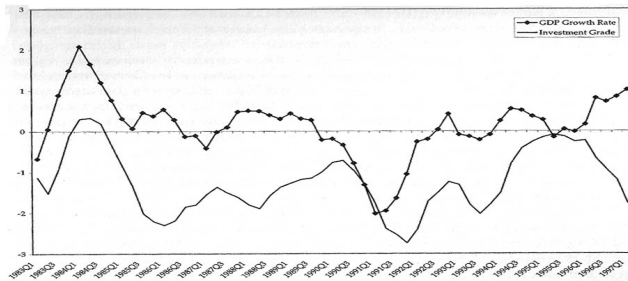
## Example 2: Distance from default conditional on macro variables (Pesaran, Schuermann)



Distance from default conditional on state of the economy

Source: Pesaran and Schuermann 2003

## Example 3: Rating Transition Matrices Depending on Macro Variables



GDP Growth Rate vs. Upgrade/Downgrade Ratio

Source: D. Duffie, K. J. Singleton, *Credit Risk*, Princeton (2003), p.89

Rating transition matrices are not constant.  
They vary with the macroeconomic cycle.



## Example 3: Rating Transition Matrices Depending on Macro Variables

- Transition matrix generated by intensity matrix  
 $\Pi(t, s) = \exp(\Lambda(s - t))$ .
- Kavvathas 2001: Intensity Matrix  
 $\Lambda_{ij}(t) = \exp(\eta_{ij} + \gamma'_{ij} X(t))$ .
- Lando 1998: Assume stochastic generator matrix  $\Lambda$ , which can be diagonalised:  $\Lambda(t) = B\mu(t)B^{-1}$   
 and for eigenvalues of  $\Lambda$ :  $\mu_j(t) = \gamma_{j0} + \gamma_{j1} \cdot X_t$  (affine dependence on a macroeconomic state process  $X_t$ .)

Warning: These double-stochastic models assume conditional independence of defaults given macroeconomic state. This is probably violated empirically (Das, Duffie, Kapadia, Saita 2005).

## Example 4: Dependence of interest rates on other macro variables

### Challenge:

Interest rates depend on markets **and** on central bank decisions taking into account the macroeconomic situation

- Quick and dirty: Take (some) interest rates as macro variables
- Nelson-Siegel yield factors dependent on macro variables (Diebold etc.)
- Include macro-variables in no-arbitrage term structure models (Rudebusch, Piazzesi, etc.)





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# Modelling Macroeconomic Data

A brief introduction to the basic options for macroeconomic models in a credit risk framework

Modern econometric modelling of the macroeconomy follows three different broad approaches:

- Large scale simultaneous equation models.
- Dynamic stochastic general equilibrium models.
- Various forms of vector autoregressive models.



# Modelling Macroeconomic Data

## Large scale simultaneous equation models

- These models go back to the 1950s. Usually developed and used by national research institutes and central banks.
- Basic building blocks: Equilibrium conditions, expectations equations and dynamic adjustment equations.
- Disadvantage for use in credit risk modelling: Very large and complex, difficult to correct for misspecification and difficult to estimate.
- Viewed with scepticism by modern macroeconometrics.

# Modelling Macroeconomic Data

## Dynamic stochastic general equilibrium models

- Based on stylized intertemporal competitive general equilibrium models. All macroeconomic relations derived from intertemporal optimizing behavior under technological and financial constraints as well as economic equilibrium conditions.
- Strongly based on modern macroeconomic theory. Today mainly used by central banks.
- Disadvantage for use in credit risk modelling: Very complex and relies on many strong assumptions about individual behavior, rationality and the working of market mechanisms. Using the models with data requires usually many approximations.

# Modelling Macroeconomic Data

## Vector Autoregression (VAR)

- Focus is on a small set of core macroeconomic variables with a particular emphasis on statistical fit. Extensively used in macroeconomic forecasting.
- Cointegration allows for long-term relations between macro-variables (e.g. PPP)
- Particularly suited for use in credit risk modelling because ideally constructed for studying the response of core variables to shocks. Easy to implement.
- Disadvantage: Stress testing requires the connection of data to economic stories and economic theory. This connection can be achieved with VARs only by adding more structure and thus greater model complexity.
- A particularly useful example of this approach is the Global VAR model by Pesaran et. al.

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# The Task

## Wanted:

- **Scenarios** from the multivariate distribution function of credit, market, and macro risk factor changes
- at a common time horizon of 3m to 1 yr  
given daily data of market variables and quarterly data of macro variables

## Two Approaches

- Approach 1:  
Estimate joint distribution and draw scenarios from estimated distribution
- Approach 2:  
Non-parametric methods to determine scenarios directly from historical data  
“Historical Simulation”



# Approach 1

Three steps:

- 1 Determine the marginal distribution of each risk factor at the common time horizon
- 2 Determine the dependence structure of risk factors by estimating the copula function
- 3 Calculate joint distribution from marginals and copula using Sklar's theorem

# Estimating the Marginals

## Five decisions

- Presence of autocorrelations?
- Do errors have constant volatility?
- Which distribution of error terms? (Normal, Student, or EVT, ...)
- Optimal Length of time window for parameter estimation?
- How to estimate distribution of market variables at long time horizon?

# Estimating the Copula

- 1 transform univariate time series to uniform with inverse distribution function estimated for marginals
- 2 Choose a parametric family of copulas e.g. Gauss, t-, grouped t-, etc.  
Against Gaussian: Zero tail dependence not plausible in crashes, if prices of several securities tend to drop sharply.
- 3 Choose member of family by estimating parameters from multivariate data (with uniform marginals)

## Approach 2

- Non-parametric methods to determine scenarios directly from historical data  
“Historical Simulation with time series of different frequencies”
- No other alternative to get scenarios from joint distribution if some risk factors have continuous distribution and others discrete distribution

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# Evaluate Partial Scenarios

Start from:

- Portfolio model: determines portfolio value in function of risk factor values
- Joint distribution of risk factors
- Partial scenario: fixes the value of some but not all risk factors

**Task:** Evaluate portfolio behaviour in scenario

# Approach 1

Extend partial scenario to full scenario

- fill in values of risk factors not specified by partial scenario:  
**Expectation Value conditional** on values of specified risk factor values

Result: unambiguous profit/loss number

Disadvantage: Disregard remaining risk of non-specified risk factors

## Approach 2

Analyse distribution conditional on specified risk factor values

- 1 full joint distribution and specified risk factor values determine conditional risk factor distribution
- 2 approximate conditional profit-loss distribution by Monte Carlo draws from conditional risk factor distribution and portfolio evaluation
- 3 characterise conditional profit-loss distribution by expectation value, expected shortfall, VaR etc.



## Illustrate Approach 2

Scenario	$\Delta$ GDP	$\Delta$ IR	Exp. P/L	Exp. Shortfall
Current Situation	$\pm 0\%$	$\pm 0$	+2%	3.5 bn
GDP stress	-5%	- -	-6.1%	6.2 bn
GDP/IR stress	-5%	-150 bp	-2%	6.8 bn

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# OeNB Systemic Risk Monitor

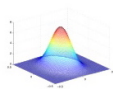
## Macro Stress testing at the Austrian Central Bank

- **Systemic Risk Monitor** (SRM) is a software developed by the Austrian Central Bank (OeNB) for systemic financial stability analysis and stress testing of banking systems.
- One of the core modules in SRM is a credit risk model where obligor pds depend explicitly on macro variables.
- While SRM focuses at a banking system rather than an individual bank the general approach might be interesting for individual banks working on macro stress testing.

# OeNB Systemic Risk Monitor

## The credit risk module

1. Draw macro-scenario  
from VAR macro model  
factor change distribution



2. Calculate obligor pds

$$\hat{\mu}_{i,s} = \frac{e^{X_t \beta_i}}{1 + e^{X_t \beta_i}}$$

3. Compute loan loss  
distribution



4. Draw loan losses



Scenario generating loop

# OeNB Systemic Risk Monitor

## Our experience in practice

- Explaining probabilities of default by macro variables alone gives a poor fit.
- Macroeconomic variables have to be supplemented by individual firm variables.
- The simulation is computationally intensive.



# OeNB Systemic Risk Monitor

## A typical stress test

- The general principle of the stress tests is based on **conditional credit portfolio loss distributions**.
- Two stress tests are considered: GDP drop by and a parallel shift in the yield curve.
- We compare the distribution of losses relative to regulatory capital under normal conditions and under stress.

# OeNB Systemic Risk Monitor

## Output of the stress test

Scenario	$\Delta$ GDP	$\Delta$ IR	Credit Risk	
			Mean	99%
Current situation	$\pm 0$	$\pm 0$	1.74	2.82
GDP stress	-2%	–	1.82	2.99
Interest rate stress	–	+120 bp	1.75	2.87

## And here are the main points again ...

### Stress Testing ...

- ... measures risk coherently (whatever your scenarios are)
- ... can be the basis of informed risk decisions if one is sure there are no worse scenarios above a plausibility level
- ... should be done systematically by identifying worst case scenarios

## And here are the main points again ...

### Credit Risk and Macro Variables

PDs, exposure at default, and collateral values depend on the state of the economy.

Risk management has to take this into account:  
Credit risk models have to encompass macroeconomic variables



## And here are the main points again ...

To perform macro stress tests ...

- ... determine macro scenarios
- ... draw values of other risk factors from joint distribution
- ... analyse the resulting conditional P/L distribution