How to find plausible, severe, and useful stress scenarios

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The Stress Testing Requirements of Basel II

- **Plausibility:** "Quantitative criteria should identify plausible stress scenarios to which banks could be exposed." (par. 718 (LXXIX)),
- **Severity:** " (...) a bank should also develop its own stress tests which it identifies as most adverse based on the characteristics of its portfolio" (par. 718 (LXXXIII))
- **Usefulness:** "Qualitative criteria should emphasize that two major goals of stress testing are to evaluate the capacity of the bank's capital to absorb potential large losses and to identify steps the bank can take to reduce its risk and conserve capital." (par. 718(LXXIX))

Plausibility

Severity

Usefulness

Application to Loan Portfolios

Outline

The framework

- Given is a portfolio of loans. The value of the portfolio is a function v of n **macro risk factors** $\mathbf{r} = (r_1, \dots, r_n)$ and of m **idiosyncratic risk factors** $\epsilon_1, \dots \epsilon_m$, one for each counterparty.
- The macro risk factor changes are distributed elliptically with covariance matrix Σ and expectations $\boldsymbol{\mu}$.The idiosyncratic risk factors may be continuous or discrete.

Plausibility of scenarios

- The plausibility of macro scenarios will be measured by the **Mahalanobis distance**:

$$\text{Maha}(\mathbf{r}) := \sqrt{(\mathbf{r}-\boldsymbol{\mu})^T \cdot \Sigma^{-1} \cdot (\mathbf{r}-\boldsymbol{\mu})},$$

  where $\mathbf{r}$, μ, and Σ only refer to the macro risk factors fixed by the scenario.
- Interpretation: Maha($\mathbf{r}$) is (the multivariate analogue of) the size of the move measured in standard deviations.

Standard stress testing with partial scenarios

Partial scenario: Specify the value of some but not all macro risk factors. What about the other risk factors?

- $\mathbf{r}_A$ other macro risk factors: **last observed value**
- $\mathbf{r}_B$ other macro risk factors: **unconditional expectation** value.
- $\mathbf{r}_C$ other macro risk factors: **conditional expected value** given the values of the fixed risk factors.
- $\mathbf{r}_D$ other macro factors **not fixed** but distributed according to the conditional distribution given the values of the fixed risk factors.

Complete partial scenario so as to maximise plausibility

Proposition 1: Assume the distribution of macro risk factors is elliptical with density strictly decreasing as a function of Maha.

$$\text{Maha}(\mathbf{r}_C) = \text{Maha}(\mathbf{r}_D)$$

This is the **maximal** plausibility which can be achieved among *all* macro scenarios which agree on the fixed risk factors.

Outline

Worst case search

- Contrary to the standard practice of stress testing we suggest to **search systematically** for those macro scenarios in some **domain of given plausibility** which are **most harmful** to the portfolio.
- By such a systematic search over an admissible domain we do not miss any harmful yet plausible scenarios.
- This is an optimization problem: **In the set**

$$\mathbf{Ell}_k := \{\mathbf{r} : \mathbf{Maha}(\mathbf{r}) \leq k\} \tag{1}$$

find the macro scenario minimizing the conditional expectation of the profit distribution.

Advantages of worst case search over standard stress testing

- Worst case scenarios are superior to the standard scenarios: **more severe** and **more or equally plausible**.
- Worst case scenarios **reflect portfolio specific dangers**.
- Worst case scenarios **allow for an identification of the key risk factors** which contribute most to the loss in the worst case scenario.

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Identify key risk factors

Key risk factors are the risk factors with the highest Maximum Loss Constribution (MLC).

$$MLC(i) := \frac{CEP(\mathbb{E}r_1, \mathbb{E}r_2, \ldots, r_i^{WC}, \mathbb{E}r_{i+1}, \ldots \mathbb{E}r_n) - CEP(\mathbb{E}\mathbf{r})}{CEP(\mathbf{r}^{WC}) - CEP(\mathbb{E}\mathbf{r})}.$$

- $MLC(i)$ is the loss if risk factor i takes its worst case value and the other risk factors take their expected values, as a percentage of MaxLoss.

Interaction of risk factors

Proposition

Assume CEP as a function of the macro risk factors has continuous second order derivatives.
The loss contributions of the risk factors add up to 100% for all scenarios $\mathbf{r}$, $\sum_{i=1}^{n} LC(i, \mathbf{r}) = 1$, *if and only if CEP is of the form*

$$CEP(r_1, \ldots, r_n) = \sum_{i=1}^{n} g_i(r_i). \quad (2)$$

This is the case if and only if all cross derivatives of CEP

$$\frac{\partial^2 CEP(\mathbf{r})}{\partial r_i \partial r_j} = 0$$

vanish identically for $i \neq j$.

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Credit risk model

- Portfolio of foreign currency loans with N obligors, one period.
- Payment obligation to the bank at time 1 in home currency is

$$o_i = l_i(1+r)\ f(1)/f(0)\ + l_i\ s\ f(1)/f(0)$$

- The payment ability of obligor i is distributed according to

$$\begin{aligned} a_i(1) &= a_i(0)\cdot \frac{GDP(1)}{GDP(0)} \cdot \epsilon, \\ \log(\epsilon) &\sim N(\mu, \sigma) \end{aligned}$$

  where m and a(0) are constants, and $\mu = -\sigma^2/2$ ensuring $E(\epsilon) = 1$. The realizations of ϵ_i are independent
- The profit bank makes with obligor i is

$$v_i := \min(a_i, o_i) - l_i(1+r)f(1)/f(0).$$

Calibrating the idiosyncratic risk distribution

- Let p_i be the annual default probability

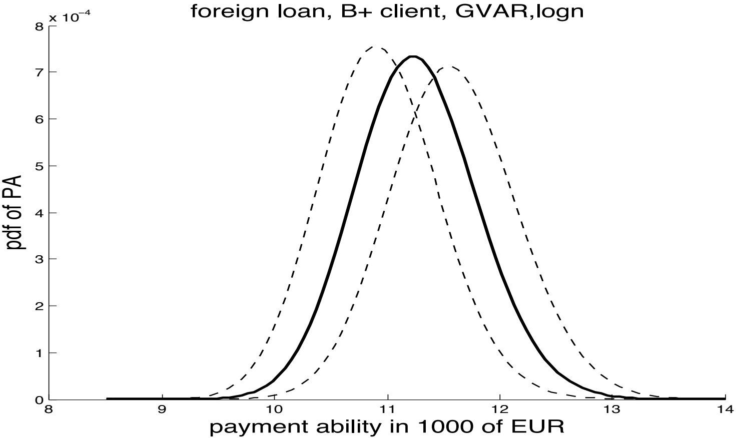
$$p_i = P[a_i(\sigma) < o_i(s)]$$

- Spreads are set to achieve some target expected profit for each loan:

$$E(v_i(\sigma, s)) = \mathrm{EP}_{\mathrm{target}},$$

  where v_i is the profit with obligor i and $\mathrm{EP}_{\mathrm{target}}$ is some target expected profit.
- The two free parameters σ and s are determined from these two conditions.

Payment ability distribution under GDP shifts



Portfolio

- Portfolio consists of 100 loans with principal $l = 10000$ Euro for each obligor.
- Loans are taken in CHF from Austrian customers in rating class $B_+ (p_i = 2\%)$ and $BBB_+ (p_i = 0.1\%)$
- $a_i(0)$=1.2 l
- Spreads are set so expected profit on a loan of 10 000 Euro is 160 Euro. This gives spreads of 158.06 bp for BBB_+ and 163.88 bp for B_+ customers.

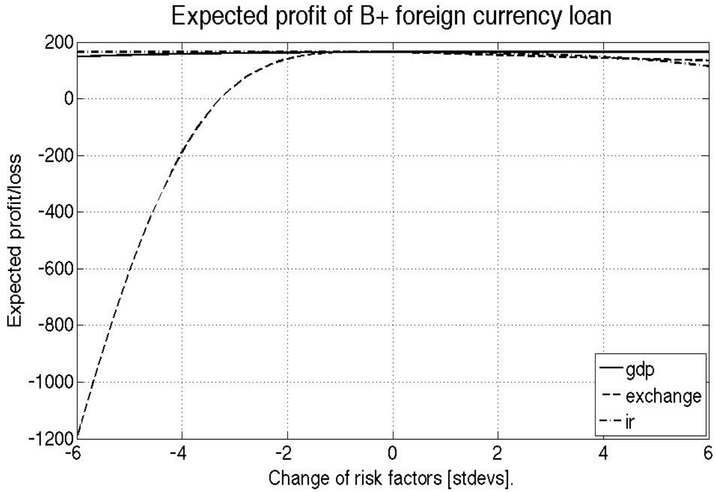
Standard Stress test versus Worst Case Analysis

scenario	Maha	CEP
foreign B+		
expected	0	16 001
GDP -3%	5.42	15 950
worst case	5.42	**-98 101**
home B+		
expected	0	16 000
GDP -3%	5.42	14 249
worst case	5.42	**13 291**

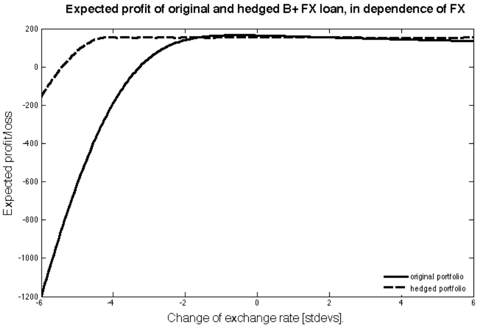
Worst Case Analysis for FX loan portfolio

	Worst Macro Scenario						
max.	GDP		foreign IR		**CHF/€**		
Maha	stdv	MLC	stdv	MLC	stdv	MLC	CEP
foreign	**B+**						
2	-0.14	0.5%	0.04	0.4%	2	**100.0%**	15 400
4	-0.1	0.1%	1.17	0.4%	-3.78	**65.3%**	-26 084
6	-0.03	0.0%	1.59	0.2%	-5.74	**77.0%**	-136 000

Key Risk factors



FX-hedged loan portfolio



type	underlying	strike	quantity
long European call	CHF/€	1.53	80
long European put	CHF/€	1.45	620
long European put	CHF/€	1.40	1000
long European put	CHF/€	1.36	1250
long European put	CHF/€	1.33	1470
long European put	CHF/€	1.30	1580

Conclusions

1. **Measure plausibility of scenarios**:
   by Maha.
2. **Partial scenarios:**
   Plausibility maximised if we set the remaining risk factors to their conditional expected values (or leave them unspecified)
3. **Maximise severeness of stress scenarios:**
   Among the macroeconomic scenarios satisfying some plausibility constraint determine the worst case scenario.
4. **Identify key risk factors:**
   Risk factors with highest MaxLoss contribution MLC.