Stress Testing for Credit Risk

Thomas Breuer
thomas.breuer@fhv.at

Stress Testing - Regulation, Application, Risk Managment
RISK Incisive Training
London, September 15, 2008

Outline

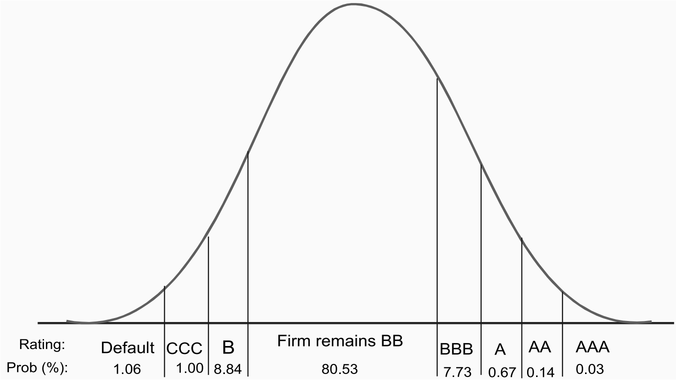
Credit risk factors

- RiskMetrics: rating of each obligor
- KMV: EDF of each obligor
- CreditRisk+: number of defaults in exposure bands
- CreditPortfolioView: macroeconomic variables in countries and sectors

Two challenges for credit stress testing result:

- discrete risk factors possible
- many risk factors possible: one for each obligor

Mapping discrete into continuous credit risk factors



Calibrate to fit default and transition proababilities

Stress Testing with Partial Scenarios

A way how to stress test models with many risk factors:

- Partial scenarios:
  specify the values of some but not all risk factors.
- e.g. "GDP will grow by 2.5%"
  "PDs in country X double" (translated into credit risk factors of model)
- What to do about other risk factors?
  How to evaluate portfolio in a partial scenario?

Evaluate Portfolio in Partial Scenarios

- Partial scenarios: For some risk factors not a value is specified, but just a distribution (conditional given values of fixed risk factors)
- In such a scenario, we have not one portfolio value but a distribution of values.
- Stress test should give a number. Stress distribution needs to be characterised by a number.
- Take e.g. CEP: conditional expected profit in partial scenario

Standard stress testing with partial scenarios

Partial scenario: Specify the value of some but not all macro risk factors. What about the other risk factors?

- $\mathbf{r}_A$ other macro risk factors: last observed value
- $\mathbf{r}_B$ other macro risk factors: unconditional expectation value.
- $\mathbf{r}_C$ other macro risk factors: conditional expected value given the values of the fixed risk factors.
- $\mathbf{r}_D$ other macro factors not fixed: distributed according to the conditional distribution given the values of the fixed risk factors.

Complete partial scenario so as to maximise plausibility

Proposition 1: Assume the distribution of macro risk factors is elliptical with density strictly decreasing as a function of Maha.

$$\mathrm{Maha}(\mathbf{r}_C) = \mathrm{Maha}(\mathbf{r}_D)$$

Among the scenarios with the given values of the fixed risk factors, these are the macro scenarios with the highest plausibility.

Plausibility of scenarios

- The plausibility of scenarios will be measured by the Mahalanobis distance:

$$\text{Maha}(\mathbf{s}_{\text{stress}}, \boldsymbol{\mu}) := \sqrt{(\mathbf{s}_{\text{stress}} - \boldsymbol{\mu})^T \cdot \Sigma^{-1} \cdot (\mathbf{s}_{\text{stress}} - \boldsymbol{\mu})}$$

  where $\mathbf{s}_{\text{stress}}$, $\boldsymbol{\mu}$, and Σ only refer to the risk factors specified by the scenario.
- Interpretation: $\text{Maha}(\mathbf{s}_{\text{stress}}, \boldsymbol{\mu})$ is (the multivariate analogue of) the size of the move measured in standard deviations.

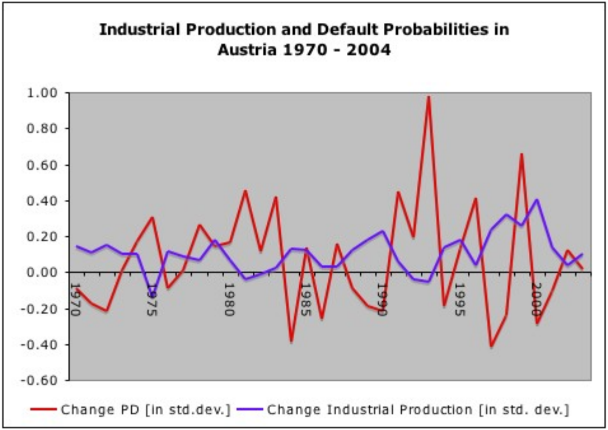
Outline

Market and Macro matters for credit risk

Many ingredients of credit risk models depend on the macroeconomic situation

- default rates and other rating transition probabilities
- recovery rates depend on collateral values (e.g. real estate and equity)
- exposure at default
  (depends often on interest or exchange rates)

Macroecomics and Credit Risk



Example 1: Credit Portfolio View (Wilson/McKinsey, Nickell et al.)

$$P_{j,t} = \frac{1}{1 + \exp\left(-Y_{j,t}\right)}$$

$$Y_{j,t} = \beta_{j,0} + \beta_{j,1} X_{j,1,t} + \beta_{j,2} X_{j,2,t} + \ldots + \beta_{j,m} X_{j,m,t} + \nu_{j,t}$$

$P_{j,t}$: cond. PD for obligor in country/industry j
$Y_{j,t}$: macroeconomic index for country/region j
β_j: coefficient vector to be estimated from default data in country/region j
$X_{j,t}$: vector of macro variables for country/region j
evolution modelled by an autoregressive model
$\nu_{j,t}$: error term

Example 2: Structural default models with macroeconomic variables (Pesaran, Schuermann)

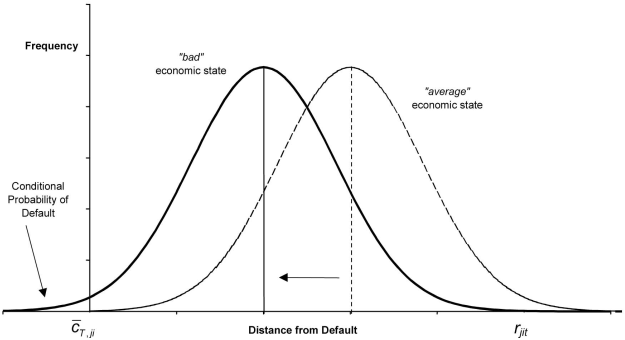
- structural model: firm (equity) value below threshold triggers default
- model firm (equity) value as

$$r_{ij,t} = \mu_{ijt} + \xi_{ij,t}$$

μ_{ijt}: forecastable conditional mean,
depending on macro variables, firm specific 'alphas',
regional fixed and time trend effects, global exogeneous variables

$\xi_{ij,t}$: error distribution, involving firm shocks,
macro shocks, exogeneous shocks

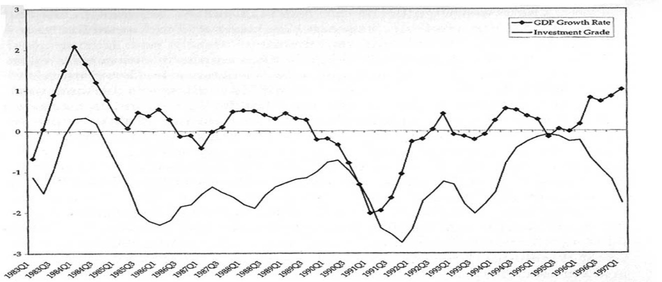
Example 2: Distance from default conditional on macro variables (Pesaran, Schuermann)



Distance from default conditional on state of the economy

Source: Pesaran and Schuermann 2003

Example 3: Rating Transition Matrices Depending on Macro Variables



GDP Growth Rate vs. Upgrade/Downgrade Ratio
Source: D. Duffie, K. J. Singleton, Credit Risk, Princeton (2003), p.89

Rating transition matrices are not constant.

They vary with the marcoeconomic cycle.

Example 3: Rating Transition Matrices Depending on Macro Variables

- Transition matrix generated by intensity matrix $\Pi(t,s) = \exp\left(\Lambda(s-t)\right)$.
- Kavvathas 2001: Intensity Matrix $\Lambda_{ij}(t) = \exp\left(\eta_{ij} + \gamma_{ij}' X(t)\right)$.
- Lando 1998: Assume stochastic generator matrix Λ, which can be diaganolised: $\Lambda(t) = B\mu(t)B^{-1}$ and for eigenvalues of Λ: $\mu_j(t) = \gamma_{j0} + \gamma_{j1} \cdot X_t$ (affine dependence on a macroeconomic state process X_t.)

Warning: These double-stochastic models assume conditional independence of defaults given macroeconomic state. This is probably violated empirically (Das, Duffie, Kapadia, Saita 2005).

Outline

Traditional Stress Tests

How to select scenarios

How to select scenarios

- Standard scenarios
- Historical scenarios
- Subjective worst case scenarios

Dangers of Traditional Stress Tests

- A stress scenario for one portfolio might be a lucky strike for another portfolio
- Stress tests with standard and historical scenarios may nourish a false illusion of safety
- Subjective worst case scenarios are often too implausible to trigger management action

Dangers of Traditional Stress Tests

But:

Stress Tests can be the basis of informed risk decisions ...

- ... if the scenarios are plausible
- ... if we are confident that there are no scenarios which are worse and more plausible

Model based stress testing: Worst case search

- In some domain of given plausibility search systematically for those scenarios which are most harmful to the portfolio.
- By such a systematic search over an admissible domain we do not miss any harmful yet plausible scenarios.
- The search can be formulated as an optimization problem: We look for macro scenarios in the set

$$\mathrm{Ell}_k := \{\mathbf{s} : \mathrm{Maha}(\mathbf{s}) \leq k\} \tag{1}$$

minimizing the conditional expectation of the profit distribution.

Advantages of worst case search over standard stress testing

- Worst case scenarios are superior to the standard stress scenarios in the sense that they are more severe and more or equally plausible.
- Worst case scenarios reflect portfolio specific dangers.
- Worst case scenarios allow for an identification of the key risk factors which contribute most to the loss in the worst case scenario.
- Knowledge of key risk factors helps in designing hedges.

Identify key risk factors

Key risk factors are the risk factors with the highest Maximum Loss Constribution (MLC).
After identifying worst case scenario $\mathbf{r}^{WC}$ calculate

$$MLC(i) := \frac{CEP(\mu_1, \mu_2, \dots, r_i^{WC}, \mu_{i+1}, \dots \mu_n) - CEP(\boldsymbol{\mu})}{CEP(\mathbf{r}^{WC}) - CEP(\boldsymbol{\mu})}.$$

(where μ_i is expected value of risk factor i)

- $MLC(i)$ is the expected loss if risk factor i takes its worst case value and the other risk factors take their expected values, as a percentage of MaxLoss.
- The MLC of the risk factors in general do not add up to 100%.

Design Hedge

- identify key risk factors
- plot behaviour of CEP in dependence of key risk factors
- construct hedge to adjust CEP when key risk factors move (assuming key risk factors can be proxied by traded instruments)

Outline

Case study: integrated credit and market stress tests

- For variable rate and FX loans, PD depends on market risk factors
- Let us look at a simple Merton-type model integrating market and credit risk
- What can stress tests tell us about the risk characteristics of the portfolios?
- How can stress tests help to design hedges?

Sample portfolio

- Portfolio of foreign/home currency variable rate loans, one period.
- Payment obligation to the bank in home currency is

$$\begin{aligned} o_i &= l_i(1 + r_{home}) + l_i\ s && \text{for home loan} \\ o_i &= l_i(1 + r_{for})\ f(1)/f(0)\ + l_i\ s\ f(1)/f(0) && \text{for FX loan} \end{aligned}$$

- The profit bank makes with obligor i is

$$\begin{aligned} v_i &:= \min(a_i, o_i) - l_i(1 + r_{home}) && \text{for home loan} \\ v_i &:= \min(a_i, o_i) - l_i(1 + r_{for}) f(1)/f(0) && \text{for FX loan} \end{aligned}$$

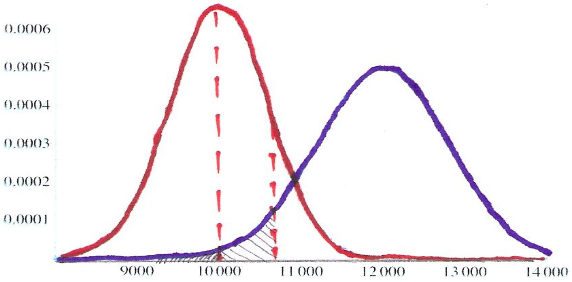
Payment ability

- The payment ability of obligor i is distributed according to

$$\begin{aligned} a_i(1) &= \frac{I_i}{LTV} \cdot \frac{GDP(1)}{GDP(0)} \cdot \epsilon, \\ \log(\epsilon) &\sim N(\mu, \sigma) \end{aligned}$$

  where LTV is initial loan-to-value ratio, and $\mu = -\sigma^2/2$ ensuring $E(\epsilon) = 1$. The realizations of ϵ_i are independent
- $\frac{I_i}{LTV}$ is initial payment ability/collateral value.
- Common exposure of obligor payment abilities to GDP give rise to correlations in PDs.

Interaction of market and credit risk



blue: stochastic payment ability (credit risk)
red: stochastic payment obligation = default barrier (market risk)
interaction: change in market risk factors can increase PD

Portfolio

- Portfolio consists of 100 loans with principal $l_i = 10000$ Euro for each obligor.
- Loans are taken in CHF from Austrian custormers in rating class $B_+(p_i = 2\%)$ and $BBB_+(p_i = 0.1\%)$
- $LTV = 83\%$
- Spreads are set so expected profit on a loan of 10 000 Euro is 160 Euro. This gives spreads of 158.06 bp for BBB_+ and 163.88 bp for B_+ customers.

Standard stress test with partial scenario

FX scenario: EUR falls by 20% against CHF.

Rating	Type	Maha	CEP
B_+	A	5.587	-64 294
B_+	B	4.979	-56 293
B_+	C	4.905	-53 337
B_+	D	4.905	-54 209

Compare to unconditional EP of +16 000.

Traditional stress test

Comparing the conditional FX stress distribution and the unconditional profit distribution

Rating	Curr	Scenario	CEP
B_+	FX	unconditional	16 000
B_+	FX	FX stress	-54 209
B_+	home	unconditional	16 000
B_+	home	FX stress	16 130

FX shock hits FX loan portfolios hard,
but not home currency loan portfolios.

Standard Stress test versus Worst Case Analysis

Rating	Curr	Scenario	Maha	CEP
B_+	FX	FX Stress	4.91	-54 209
B_+	FX	Worst Case	4.91	-68 023
BBB_+	FX	FX Stress	4.91	-45 136
BBB_+	FX	Worst Case	4.91	-62 139

Worst Case of same plausibility is worse than hand-picked FX stress.

Worst Case Analysis

	Worst Macro Scenario						
max.	GDP		IR		FX		
Maha	abs.	stdv	abs.	stdv	abs.	stdv	CEP
FX B+							
3	231.10	-0.10	0.035	0.93	1.363	-2.81	1 579
4	230.95	-0.11	0.039	1.17	1.306	-3.78	-26 138
5	230.85	-0.09	0.042	1.39	1.249	-4.75	-73 288

Compare to unconditional EP of +16 000.

Large plunges in FX are most dangerous.

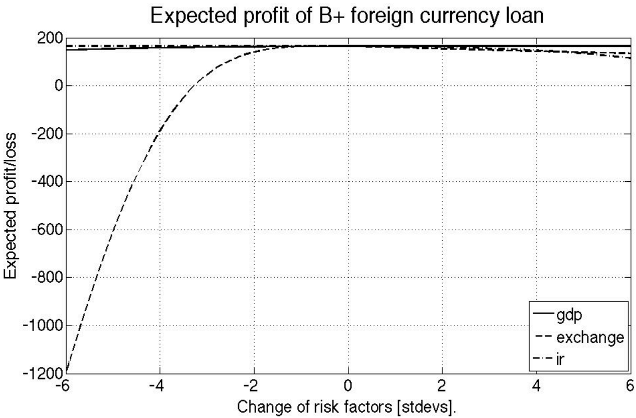
Maximum Loss Contributions

max. Maha	single factor moves GDP	IR	FX	moves of pairs GDP, IR	GDP, FX	IR, FX
FX B+						
3	0.1%	0.8%	60.0%	1.1%	66.5%	91.6%
4	0.0%	0.4%	65.4%	0.5%	70.9%	93.2%
5	0.0%	0.2%	71.1%	0.3%	75.6%	95.0%

FX alone explains only 2/3 of Maximum Loss.
FX and IR together explain more than 90%.

Outline

Key Risk factors

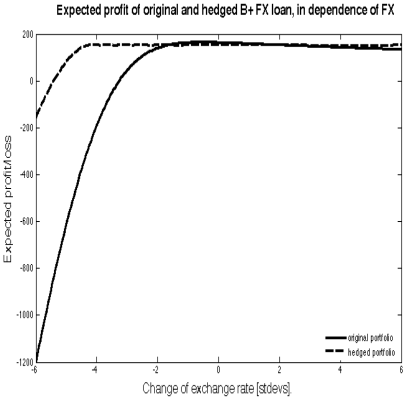


FX rate is key risk factor. Try to hedge exposure to key risk factor.

Hedge against FX moves up to $\pm 4\sigma$

type	underlying	strike	quantity
long European call	CHF/€	1.53	80
long European put	CHF/€	1.45	620
long European put	CHF/€	1.40	1000
long European put	CHF/€	1.36	1250
long European put	CHF/€	1.33	1470
long European put	CHF/€	1.30	1580

Price of hedge: €8.5



Worst Case Analysis of Hedged Loan Portfolio

max.	Worst Case Scenario						
	GDP		IR		FX		
Maha	abs.	stdv	abs.	stdv	abs.	stdv	CEP
FX B+	**hedged**						
3	230.21	-0.38	0.048	1.74	1.397	-2.26	7 124
4	229.56	-0.56	0.058	2.42	1.360	-2.91	-6 135
5	229.43	-0.54	0.062	2.72	1.299	-3.95	-27 651
FX B+	**original**						
3	231.10	-0.10	0.035	0.93	1.363	-2.81	1 579
4	230.95	-0.11	0.039	1.17	1.306	-3.78	-26 138
5	230.85	-0.09	0.042	1.39	1.249	-4.75	-73 288

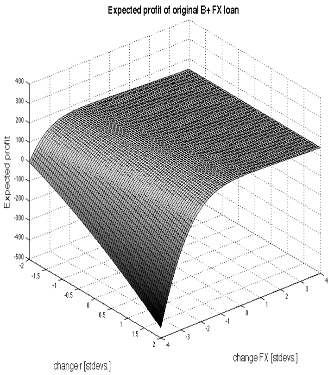
Worst case loss strongly improved, but not completely. Why?

Maximum Loss Contributions in Hedged Loan Portfolio

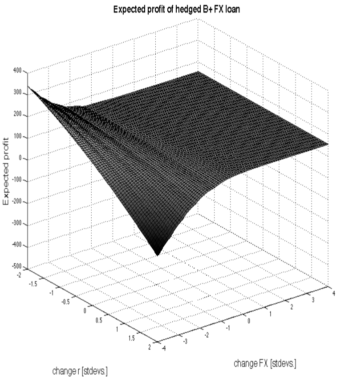
max.	single factor moves			moves of pairs		
Maha	GDP	IR	FX	GDP, IR	GDP, FX	IR, FX
B+	**hedged**					
3	0.6%	3.6%	4.9%	5.5%	19.9%	73.7%
4	0.4%	2.5%	1.9%	4.0%	17.3%	73.0%
5	0.2%	1.6%	1.1%	2.5%	17.7%	76.9%
FX B+	**original**					
3	0.1%	0.8%	60.0%	1.1%	66.5%	91.6%
4	0.0%	0.4%	65.4%	0.5%	70.9%	93.2%
5	0.0%	0.2%	71.1%	0.3%	75.6%	95.0%

MaxLoss contribution of FX strongly reduced.
But FX and IR together still contribute most to MaxLoss.
Hedge works nicely only at one level of IR.

Conditional hedge required



original



hedged

Conclusions

1. Measure plausibility of scenarios:
   by Maha.
2. Partial scenarios:
   Plausibility maximised if we set the remaining risk factors to their conditional expected values (or leave them unspecified) and consider conditional risk factor distribution
3. Maximise severeness of stress scenarios:
   Among the macroeconomic scenarios satisfying some plausibility constraint determine the worst case scenario, which has lowest CEP.
4. Identify key risk factors:
   Risk factors with highest MaxLoss contribution MLC.
5. Design hedge to adjust CEP when key risk factors move.