

Macro Stress Testing and Worst Case Analysis of Loan Portfolios

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Macro Stress Testing Methods

- Plausibility of scenarios

- Standard stress testing with partial scenarios

- Worst Case Search

- Identify key risk factors

Application to Foreign Currency Loan Portfolios



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- ▶ The present paper is an attempt to suggest a methodology that gives a precise operational definition to what it means to find "exceptional but plausible" scenarios in macro stress testing of a portfolio.
- ▶ The example portfolio which we use and develop in the paper is a portfolio of foreign currency loans to private households.

Outline

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The abstract framework

- ▶ Given is a portfolio of loans. The value of the portfolio is a function v of n **macro risk factors** $\mathbf{r} = (r_1, \dots, r_n)$ and of m **idiosyncratic risk factors** $\epsilon_1, \dots, \epsilon_m$, one for each counterparty.

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- ▶ The macro risk factor changes are distributed elliptically with covariance matrix Σ and expectations μ . The idiosyncratic risk factors may be continuous or discrete.

Plausibility of scenarios

- ▶ The plausibility of macro scenarios will be measured by the **Mahalanobis distance**:

$$\text{Maha}(\mathbf{r}) := \sqrt{(\mathbf{r} - \boldsymbol{\mu})^T \cdot \boldsymbol{\Sigma}^{-1} \cdot (\mathbf{r} - \boldsymbol{\mu})},$$

where \mathbf{r} , $\boldsymbol{\mu}$, and $\boldsymbol{\Sigma}$ only refer to the macro risk factors fixed by the scenario.

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- ▶ Interpretation: $\text{Maha}(\mathbf{r})$ is (the multivariate analogue of) the size of the move measured in standard deviations.

Standard stress testing with partial scenarios

Partial scenario: Specify the value of some but not all macro risk factors. What about the other risk factors?

- r_A Fixed risk factors: value specified by the partial scenario, other macro risk factors: **last observed value**
- r_B fixed risk factors: value specified by the partial scenario, other macro risk factors: **unconditional expectation** value.
- r_C fixed risk factors: value specified by the partial scenario, other macro risk factors: **conditional expected value** given the values of the fixed risk factors.
- r_D fixed risk factors: value specified by the partial scenario, other macro factors **not fixed**: distributed according to the marginal distribution given the values of the fixed risk factors.

Complete partial scenario so as to maximise plausibility

Proposition 1: Assume the distribution of macro risk factors is elliptical with density strictly decreasing as a function of Maha.

$$\text{Maha}(\mathbf{r}_C) = \text{Maha}(\mathbf{r}_D)$$

Among the scenarios with the given values of the fixed risk factors, these are the macro scenarios with the **highest plausibility**.

Complete partial scenario so as to maximise harm

We measure the harmfulness of a stress scenario by looking at **conditional expected profits** (CEP).

Proposition 2: If the portfolio value function v is concave in the non-fixed macro risk factors, then

$$CEP(\mathbf{r}_D) \leq CEP(\mathbf{r}_C).$$

If v is convex in the non-fixed risk factors the opposite inequality holds. If v is neither concave nor convex $CEP(\mathbf{r}_D)$ may be higher or lower than $CEP(\mathbf{r}_C)$.

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- ▶ By such a systematic search over an admissible domain we do not miss any harmful yet plausible scenarios.
- ▶ The search can be formulated as an optimization problem:
We look for macro scenarios in the set

$$\text{Ell}_k := \{\mathbf{r} : \text{Maha}(\mathbf{r}) \leq k\} \quad (1)$$

minimizing the conditional expectation of the profit distribution.



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- ▶ Worst case scenarios **reflect portfolio specific dangers**.
- ▶ Worst case scenarios **allow for an identification of the key risk factors** which contribute most to the loss in the worst case scenario.

Identify key risk factors

Key risk factors are the risk factors with the highest Maximum Loss Contribution (MLC).

$$MLC(i) := \frac{CEP(\mathbb{E}r_1, \mathbb{E}r_2, \dots, r_i^{WC}, \mathbb{E}r_{i+1}, \dots, \mathbb{E}r_n) - CEP(\mathbb{E}r)}{CEP(r^{WC}) - CEP(\mathbb{E}r)}.$$

- ▶ $MLC(i)$ is the loss if risk factor i takes its worst case value and the other risk factors take their expected values, as a percentage of MaxLoss.
- ▶ The MLC of the risk factors in general do not add up to 100%.

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Credit risk model

- ▶ Portfolio of foreign currency loans with N obligors, one period.
- ▶ Payment obligation to the bank at time 1 in home currency is

$$o_i = l_i(1+r) f(1)/f(0) + l_i s f(1)/f(0)$$

- ▶ The payment ability of obligor i is distributed according to

$$a_i(1) = a_i(0) \cdot \frac{GDP(1)}{GDP(0)} \cdot \epsilon,$$

$$\log(\epsilon) \sim N(\mu, \sigma)$$

where m and $a(0)$ are constants, and $\mu = -\sigma^2/2$ ensuring $E(\epsilon) = 1$. The realizations of ϵ_i are independent

- ▶ The profit bank makes with obligor i is

$$v_i := \min(a_i, o_i) - l_i(1+r)f(1)/f(0).$$



Calibrating the idiosyncratic risk distribution

- ▶ Let p_i be the annual default probability

$$p_i = P[a_i(\sigma) < o_i(s)]$$

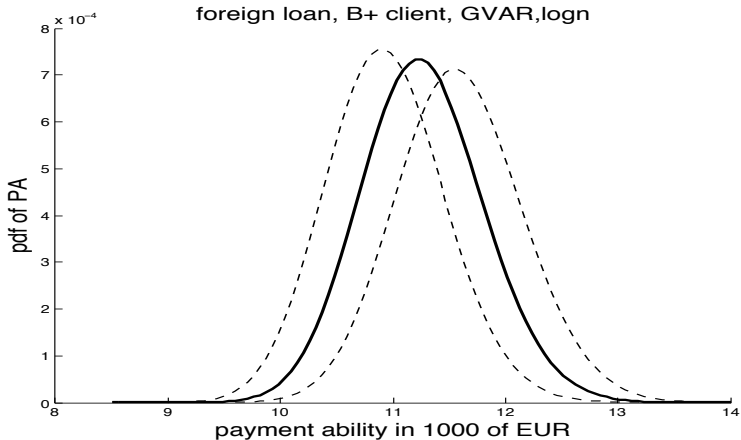
- ▶ Spreads are set to achieve some target expected profit for each loan:

$$E(v_i(\sigma, s)) = EP_{\text{target}},$$

where v_i is the profit with obligor i and EP_{target} is some target expected profit.

- ▶ The two free parameters σ and s are determined from these two conditions.

Payment ability distribution under GDP shifts



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- ▶ $a_i(0) = 1.2 I$
- ▶ Spreads are set so expected profit on a loan of 10 000 Euro is 160 Euro. This gives spreads of 158.06 bp for BBB_+ and 163.88 bp for B_+ customers.

Standard stress test with partial scenario

FX scenario: EUR falls by 20% against CHF.

Rating	Type	Maha	CEP
<i>B</i> ₊	A	5.587	-64 294
<i>B</i> ₊	B	4.979	-56 293
<i>B</i> ₊	C	4.905	-53 337
<i>B</i> ₊	D	4.905	-54 209
<i>BBB</i> ₊	A	5.587	-58 134
<i>BBB</i> ₊	B	4.979	-48 225
<i>BBB</i> ₊	C	4.905	-44 587
<i>BBB</i> ₊	D	4.905	-45 136

Compare to unconditional EP of +16 000.

Standard Stress test versus Worst Case Analysis

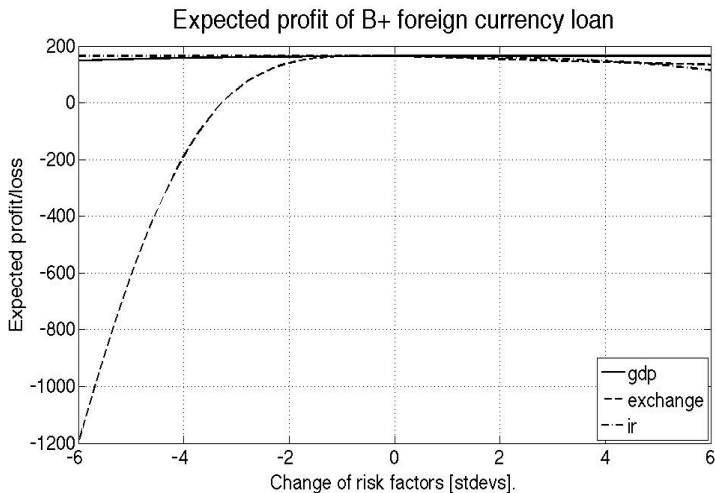
Rating	Scenario	Maha	CEP
B_+	Stress	4.91	-53 337
B_+	Worst Case	4.91	-68 023
BBB_+	Stress	4.91	-44 587
BBB_+	Worst Case	4.91	-62 139

Worst Case Analysis

Worst Macro Scenario

max. Maha	GDP		foreign IR		CHF/€		CEP
	stdv	MLC	stdv	MLC	stdv	MLC	
foreign	B+						
2	-0.14	0.5%	0.04	0.4%	2	100.0%	15 400
4	-0.1	0.1%	1.17	0.4%	-3.78	65.3%	-26 084
6	-0.03	0.0%	1.59	0.2%	-5.74	77.0%	-136 000
foreign	BBB+						
2	-0.14	0.0%	0.03	0.0%	2	100.0%	14 855
4	-0.27	0.0%	0.07	0.0%	4	100.0%	13 859
6	-0.04	0.0%	1.58	0.0%	-5.74	75.8%	-135 203

Key Risk factors



Conclusions

1. **Measure plausibility of scenarios:**
by Maha.
2. **Partial scenarios:**
Plausibility maximised if we set the remaining risk factors to their conditional expected values (or leave them unspecified)
3. **Maximise severeness of stress scenarios:**
Among the macroeconomic scenarios satisfying some plausibility constraint determine the worst case scenario.
4. **Identify key risk factors:**
Risk factors with highest MaxLoss contribution MLC.