Risk Capital for Interacting Market and Credit Risk: Variable Rate Loans

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1. The myth of ‘diversification’ between risk types
2. Integrated versus separate analysis of market and credit risk
3. Example: Variable Rate Loans

Outline

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Regulation of Market and Credit Risk: The traditional view.

- In the work of the Basel Commitee there has been a tradition to distinguish market from credit risk and to treat both categories independently in the calculation of risk capital.
- Leaving aside operational risk, Pillar 1 of Basle II requires separate regulatory capital for market and credit risk:

$$RC_c + RC_m \tag{1}$$

- In this paper we argue that this approach is problematic and that it can lead to significant underestimation of risk.

The Argument for 'Diversification' Benefits between Market and Credit Risk

Premise 1 'Diversification': Under a subadditive risk measure the risk of the total portfolio will be smaller or at most equal to the sum of the risk of the banking book and of the trading book.

Premise 2 Credit risk is just relevant to the banking book and market risk is just relevant to the trading book.

Conclusion Under all subadditive risk measures total risk will be smaller or at most equal to the sum of market risk and credit risk.

This argument is valid.
We criticise Premise 2 and the Conclusion.

Related Research

- Literature that takes a critical view on the traditional risk categorization and concentrating on modelling issues: Jarrow and Turnbull [2000], Medova and Smith [2005], Barnhill and Maxwell [2002].
- Literature that does not take issue with the traditional categorization but looks at portfolios subject to market- and credit risk as subportfolios of the overall bank portfolio: Rosenberg and Schuermann [2006] and Dimakos and Aas [2004].

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A formalization of separated and integrated risk analysis

- Assume some separation of risk factors into market risk factors described by a vector $e \in E$ and credit risk factors, described by a vector $a \in A$ is given.
- The value of the portfolio in dependence of a and e is given by the function $v : A \times E \to \mathbb{R}$.
- Market risk is defined as the value change of a portfolio which arises from moves in market risk factors, assuming that credit risk factors are constant at some a_0:

$$\Delta m(e) := v(a_0, e) - v(a_0, e_0).$$

- Credit risk deals with value changes caused by moves in credit risk factors, assuming all market risk factors are constant at e_0:

$$\Delta c(a) := v(a, e_0) - v(a_0, e_0).$$

- Integrated risk is the value change caused by simultaneous moves of market and credit risk factors:

$$\Delta v(a, e) := v(a, e) - v(a_0, e_0).$$

Approximation of regulatory capital by adding up categories

In a separate analysis of market and credit risk regulatory capital for market and credit risk are simply added up. This amounts to assuming that integrated risk is approximately the sum of market risk plus credit risk:

$$\Delta v(a,e) \approx \Delta c(a) + \Delta m(e).$$

This corresponds to the approximation

$$v(a,e) \approx v(a_0,e_0) + \Delta c(a) + \Delta m(e) =: \hat{v}(a,e).$$

Approximation errors can go both ways

Our main message is that the approximation $\Delta c(a) + \Delta m(e)$ not always overestimates but sometimes underestimates the true integrated Δv. If in some scenario (a, e) the approximation error

$$\begin{aligned} d(a,e) &:= \Delta v(a,e) - \Delta c(a) - \Delta m(e) \\ &= v(a,e) - \hat{v}(a,e) \end{aligned}$$

is negative, we have a malign risk interaction. If d is non-negative in all scenarios, we have a benign interaction effect.

A classification of portfolios where the approximation is exact

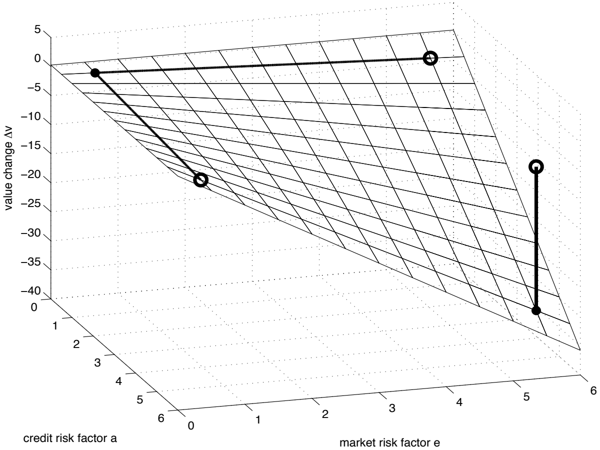
Proposition

The approximation is exact, that is $\Delta v(a, e) = \Delta c(a) + \Delta m(e)$, *if and only if* v *has the form*

$$v(a, e) = v_1(a) + v_2(e).$$

for some functions v_1, v_2.
In this case the portfolio is separable into two subportfolios, one depending only on credit risk factors, the other depending only on market risk factors.

Separate analysis may underestimate total losses



Measuring the effects of integrated risk analysis

- Given a distribution on the space (a, e) of scenarios these properties of the value function v carry over to profit distributions:
  For a (total, market, credit) profit distribution X risk capital is

  $$\mathrm{RC}_\alpha(X) := E(X) - \mathrm{ES}_\alpha(X), \tag{2}$$

  where ES_α is Expected Shortfall at some confidence level α
- We measure the integration effect by an index:

  $$I := \frac{RC(\Delta v)}{RC(\Delta c) + RC(\Delta m)}$$

  I measures the percentage amount by which total risk exceeds the sum of market and credit risk.
  $I = 1$: neither positive nor negative integration effects.

Outline

How do market and credit risk interact?

- PD may depend on market risk factors
  adjustable rate mortgages: interest rate increase drives up PD.
  carry trades: FX rate increase drives up PD
- EAD depends on market risk factors
- LGD depends on market risk factors
  subprime mortgage: collateral real estate loses value
- Feedback from PD to collateral
  Increased defaults trigger fire sales of collateral
  and market prices of collateral fall
- Defaults increase exposure to market risk
  by deleting one side of hedge

Can risk underestimation matter in a real world example?

- Portfolio of variable rate loans with N obligors indexed by $i = 1, \ldots, N$, one period of four quarters, refinanced on the interbank market
- Customer's payment obligation to the bank at time 1 is

$$o_i = l_i(1 + r) \; + l_i \; s \, .$$

  where r is average interest rate over four quarters and s is spread earned by bank

- Let obligor $i's$ payment ability be a_i. Bank profit with with obligor i is

$$v_i := \min(a_i, o_i) - l_i(1 + r)$$

Payment Ability

- The payment ability of obligor i is distributed according to

$$
\begin{aligned}
a_i &= \frac{l_i}{LTV}\left(\beta_i\left(\frac{GDP(1)}{GDP(0)}-1\right)+1\right)\cdot\epsilon \\
log(\epsilon) &\sim N(\mu,\sigma)
\end{aligned}
$$

  where m and LTV are constants, and $\mu = -\sigma^2/2$ ensuring $E(\epsilon) = 1$. For different obligors the realizations of ϵ_i are independent
- LTV: initial loan-to-value-ratio
- β_i: elasticity of payment ability changes to GDP changes. Common dependence on GDP causes correlation of payment ability changes $\Delta a_i, \Delta a_j$ of obligors i, j:

$$
\mathrm{Corr}(\Delta a_i, \Delta a_j) = \beta_i\beta_j \frac{\mathrm{Var}(\Delta GDP)}{\sqrt{\mathrm{Var}(\Delta a_i)\mathrm{Var}(\Delta a_j)}}.
$$

Calibrating the integrated risk model to a rating system

- Let p_i be the annual default probability

$$p_i = P[a_i(\sigma) < o_i(s)]$$

- Spreads are set to achieve some target expected profit for each loan:

$$E(v_i(\sigma, s)) = \mathrm{EP}_{\mathrm{target}},$$

  where v_i is the profit with obligor i and $\mathrm{EP}_{\mathrm{target}}$ is some target expected profit.
- The two free parameters σ and s are determined from these two conditions.

rating	LTV	σ	spread [bp]
B+	0.17	0.73	205.1
B+	0.25	0.58	197.2
B+	0.50	0.30	181.1
B+	0.83	0.07	165.6
B+	0.91	0.03	162.5

Portfolio and Monte Carlo Simulation

- $N = 100$ loans of $l_i = 10\,000$ EUR taken out in CHF by customers in the rating class B+ ($p_i = 2\%$), or in rating class BBB+ ($p_i = 0.1\%$).
- LTV=25% in rating class BBB+, LTV=50% in rating class B+.
- Profit distribution calculated by a Monte Carlo simulation of 100 000 draws from the distribution of market and macro risk factors $GDP(1)$, and r. In each macro scenario defaults of the customers' payment abilities were determined by draws from the distribution of the payment ability process.

Expected Shortfall of different risk types for the variable rate loans.

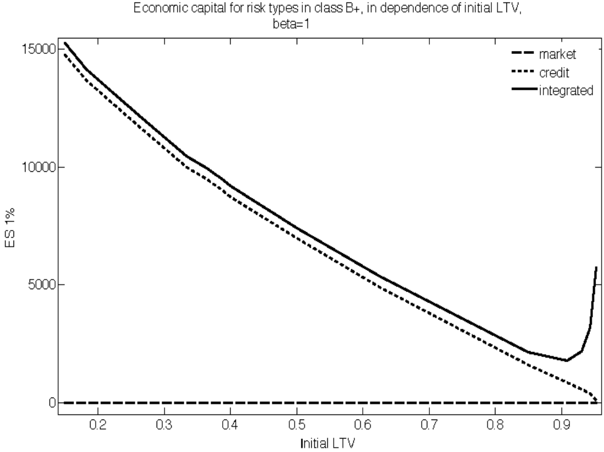
α	(a) MR, no CR RC(ΔM)		(b) CR, no MR RC(ΔC)		(c) MR&CR RC(ΔV)		c/(a+b) I
BBB+							
10%	0	(0)	862	(14)	979	(15)	1.14
5%	0	(0)	1588	(20)	1721	(20)	1.08
1%	0	(0)	3042	(35)	3187	(36)	1.05
0.1%	0	(0)	4675	(87)	4859	(92)	1.04
B+							
10%	0	(0)	3906	(24)	4178	(26)	1.07
5%	0	(0)	4851	(31)	5177	(32)	1.07
1%	0	(0)	6863	(59)	7301	(63)	1.06
0.1%	0	(0)	9679	(171)	10319	(168)	1.07

Higher LTV and lower σ lead to more dangerous compounding effect

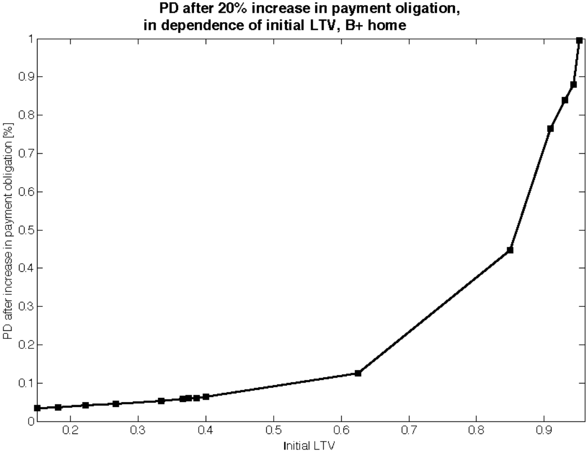
α	(a) MR, no CR RC(ΔM)		(b) CR, no MR RC(ΔC)		(c) MR&CR RC(ΔV)		c/(a+b) I
0.17	0	(0)	14059	(113)	14545	(116)	1.03
0.25	0	(0)	11896	(100)	12345	(103)	1.04
0.50	0	(0)	6863	(59)	7301	(63)	1.06
0.83	0	(0)	1810	(17)	2334	(22)	1.29
0.91	0	(0)	835	(10)	1772	(22)	2.12
0.95	0	(0)	96	(13)	5721	(149)	59.45

ØNB

Higher LTV and lower σ lead to more dangerous compounding effect



Market risk enhances credit risk more violently if initial LTV is high and σ is low.



Conclusions

- The traditional approach of treating market and credit risk separately in current regulation is problematic because many portfolios are not separable along these categories.
- Current practice of determining regulatory capital is conservative only if such subportfolios can be constructed.
- If portfolio positions depend simultaneously on both market and credit risk factors a separation into subportfolios of market and credit risk leads to wrong valuation and as a consequence to wrong assessment of the true portfolio risk.
- The approach of treating market and credit risk separately for the calculation of regulatory capital should be reconsidered.